

Variations of nuclear binding with quark masses

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We investigate the variation with light quark mass of the mass of the nucleon as well as the masses of the mesons commonly used in a one-boson-exchange model of the nucleon-nucleon force. Care is taken to evaluate the meson mass shifts at the kinematic point relevant to that problem. Using these results, the corresponding changes in the energy of the 1S_0 anti-bound state, the binding energies of the deuteron, triton and selected finite nuclei are evaluated using a one-boson exchange model. The results are discussed in the context of possible corrections to the standard scenario for big bang nucleosynthesis in the case where, as suggested by recent observations of quasar absorption spectra, the quark masses may have changed over the age of the Universe.

I. INTRODUCTION

In the last decade there has been considerable interest in the possibility that the fundamental “constants” of Nature may actually change with time [1]. Although it remains controversial, there is growing evidence that the fine structure constant may have varied by an amount of order a few parts in 10^{-5} over a period of 5–10 billion years [2–5]. It has even been suggested that this variation may have a dipole structure as we look back in different directions [6]. Although this possible variation is quite small, within the framework of most attempts at grand unification, a variation of α implies considerably larger percentage changes in quantities such as Λ_{QCD} and in the quark masses [7–9]. For example, in Ref. [7] it was shown that the variation $\delta m_q/m_q$ would be of order 38 times that of $\delta\alpha/\alpha$.

In the light of these developments it is very natural to ask what other signatures there may be for such changes. These may, for example, be the consequent changes in hadron masses or magnetic moments [10–12]. Indeed, in some cases the level of precision possible in modern atomic, molecular and optical physics means that it may even be feasible to detect the minute variations expected under the hypothesis of linear variation until the present day over a period as short as a year [13–15].

Another consequence of a variation in the parameters relevant to hadron structure is the possibility of observable consequences in big bang nucleosynthesis (BBN) or other nuclear phenomena such as the composition of the ash of long extinct natural nuclear reactors [16–18]. In this context, the effect of quark mass changes on the nucleon-nucleon force has been studied in effective

field theory [19, 20], most recently including constraints from lattice QCD [21, 22]. For the moment these lattice studies are at too high a quark mass to provide an accurate constraint [22, 23]. In an alternative approach based on a more traditional model of the nucleon-nucleon force, the latest work of Flambaum and Wiringa [24] on this topic involved the study of the variation of nuclear binding with quark mass using the Argonne potential and Schwinger-Dyson estimates of the variation of meson masses.

In this work we employ a one boson-exchange (OBE) model of the nuclear force to calculate the variation with changes in the quark mass of the binding energies of selected finite nuclei as well as the energy of the 1S_0 anti-bound state and the binding energies of the deuteron and triton. Apart from its intrinsic interest, this approach complements the work of Ref. [24] and a comparison of the two provides one way to gauge the possible model dependence of the variations reported. The method used here involves a detailed study of the variation of the mass of each of the mesons usually employed in a one-boson-exchange (OBE) picture of the nucleon-nucleon (NN) force. Care is taken to estimate this shift at the relevant kinematic point, not just at the real, on-shell meson mass or its pole position. These changes are then introduced into the quark-meson coupling (QMC) model for some light nuclei and a typical OBE model for the two nucleon systems and a Faddeev calculation of the triton.

In section II we examine, in turn, each of the mesons σ_0 , σ_1 , ω , ρ , π and η . Section III presents results for finite nuclei, while the two nucleon system and triton are discussed in section IV. The final section is reserved for

some concluding remarks.

II. MESON MASSES

In order to find the variations in the mass of the exchanged bosons in the nuclear interaction (σ , ρ , and ω) with respect to changes in the mass of the pion m_π , we use three ideas. First we introduce a description of the bare mass of the σ ($m_\sigma^{(0)}$) in terms of m_π using the Nambu-Jona-Laisino (NJL) model [25, 26]. Second, we introduce the contribution from the self energies for σ , ρ , and ω . Finally, we include these self energies in two different ways: for the σ the loop diagram is fixed in such a way that the total propagator contains a pole on the second sheet of the complex energy plane at the position found by Leutwyler *et al.* [27]; for the ρ and ω we use the chiral fit to partially quenched data from lattice QCD developed by Armour *et al.* [32]. Finally, through the Gell-Mann-Oakes-Renner (GMOR) relation we relate those changes to variations in the quark masses.

A. VARIATION IN m_σ WITH m_q

In this work we choose to parametrize the intermediate range attraction in the nucleon-nucleon (NN) force in terms the exchange of a σ meson, following the traditional one-boson-exchange (OBE) approach. Earlier work on the effect of changes in quark masses by Flambaum and Wiringa [24], used explicit two-pion exchange for this purpose. Almost certainly the reality is somewhere in between these extremes and a comparison between our results and those of Ref. [24] should serve to pin down the uncertainties in this sector of the calculation.

The existence of the σ meson has been somewhat controversial, largely because its width is comparable with its mass. However, a careful dispersion relation treatment using the Roy equation has served to accurately locate a pole which can be unambiguously identified with the σ meson. Of course, because of the large imaginary part of the energy of this pole, one cannot easily relate the position of the pole to the position of a bump in the $\pi\pi$ cross section. When it comes to the mass of the virtual σ meson exchanged in a OBE NN potential, it is a third value that is of interest. Indeed, the invariant mass of a meson exchanged in a typical NN interaction is very near zero and so we actually need the sigma mass for $p^2 \sim 0$. This is most readily found within an effective Lagrangian approach.

Our model for the σ meson involves a “bare” σ meson coupled to two pions. When required, the variation of the mass of this bare state with quark mass will be calculated within the NJL model. In this approach, the propagator of the dressed σ is described as a bare scalar propagator plus an infinite series of contributions of the form shown in Fig. 1. Calling this self-energy $\Sigma_{\pi\pi}^\sigma$, the

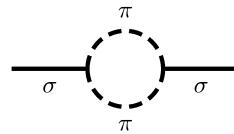


FIG. 1. Self-energy contributions for the σ meson.

total propagator can be written as:

$$\Delta_\sigma = \frac{i}{p^2 - (m_\sigma^{(0)})^2} + \frac{i}{p^2 - (m_\sigma^{(0)})^2} (i\Sigma_{\pi\pi}^\sigma) \frac{i}{p^2 - (m_\sigma^{(0)})^2} + \dots,$$

which resums to

$$\Delta_\sigma = \frac{i}{p^2 - (m_\sigma^{(0)})^2 + \Sigma_{\pi\pi}^\sigma}. \quad (1)$$

The pole in Δ_σ is the mass of the σ resonance. This pole was calculated by Leutwyler *et al.* using the method of Roy equations, which is model independent [28]. They obtained a pole located in the complex second sheet for p at

$$p = m_\sigma - \frac{i}{2}\Gamma = 441_{-8}^{+16} - i272_{-12.5}^{+9} \text{ MeV}. \quad (2)$$

The real part of the position of the resonance, $m_\sigma = 441$ MeV, is in the range (400 – 1200) MeV given by the Particle Data Group (PDG) [30], while its width, $\Gamma = 544$ MeV, is within the range (600 – 1000) MeV, also from the PDG.

Having a reliable value for the σ pole we can find a relation that lets us fix $\Sigma_{\pi\pi}^\sigma$ such that

$$\sqrt{(m_\sigma^{(0)})^2 - \Sigma_{\pi\pi}^\sigma(m_\sigma^2)} \simeq 441 - i272 \text{ MeV}. \quad (3)$$

With derivative coupling of the bare σ to two pions (consistent with chiral symmetry), the expression for the $\pi\pi$ self-energy is found to be:

$$i\Sigma_{\pi\pi}^\sigma = \frac{3}{2}\gamma_0^2 \int \frac{d^4k}{(2\pi)^4} \frac{[k^\mu (p-k)_\mu]^2}{(k^2 - m_\pi^2)((p-k)^2 - m_\pi^2)}, \quad (4)$$

where k represents the pion loop momentum, p the σ momentum and γ the $\sigma\pi\pi$ coupling (initially we took the value γ_0 from Harada, Sannino, and Schechter [29]). We are considering all these particles as elementary, so this is just an effective theory, and like any other effective theory it has to be regularized. The regularization scheme we choose is to impose a dipole cut-off (at each vertex) on the loop momentum with mass Λ :

$$\left[1 - \frac{(\frac{p}{2} - k)^2}{\Lambda^2}\right]^{-4}, \quad (5)$$

TABLE I. Parameters fixed to reproduce the position of the σ meson pole ($\gamma_0 = 6.416 \times 10^{-3} \text{ (MeV}^{-1}\text{)}$). (Δm_σ is the deviation of the fitted from the empirical value.)

$\gamma (\times \gamma_0)$	$\Lambda \text{ (MeV)}$	$m_\sigma^{(0)} \text{ (MeV)}$	$\Delta m_\sigma \text{ (MeV)} \times 10^{-5}$
4.56	320.00	563.54	$1.2 - 1.0i$
4.60	330.00	600.23	$1.1 - 0.5i$
4.70	340.00	639.85	$1.2 - 0.0i$
4.83	350.00	683.46	$1.2 - 0.0i$
5.02	360.00	732.00	$0.0 + 0.0i$
5.27	370.00	790.18	$0.9 - 1.2i$
5.61	380.00	859.15	$0.4 - 0.9i$
6.07	390.00	945.43	$0.9 - 0.9i$

which is sufficient to ensure convergence. This dipole regulator contains simple poles in k (after writing it in the form of derivatives with respect to Λ), which permits us to use contour integration over the time component.

For the remaining integral over the three-momentum we rotated \vec{k} in the complex plane $|\vec{k}|e^{i\theta}$, with $-\frac{3\pi}{2} < \theta < 0$, to ensure that the imaginary part is located in the complex second Riemann sheet. We also performed a numerical integration with the help of the routine *NIntegral* of *Mathematica*. The final value of $\Sigma_{\pi\pi}^\sigma(p^2)$ depends on two parameters: the regularization mass Λ , and the coupling constant γ_0 . We choose a range of values for Λ such that, after fixing γ_0 and $m_\sigma^{(0)}$ to reproduce the pole position (Eq. (2)), $m_\sigma^{(0)}$ varies from 560 to 950 MeV. The results are summarized in Table I, where Δm_σ represents the deviation of our result for the pole position from that of Leutwyler and collaborators.

We then define $m_\sigma^2(OBE) = (m_\sigma^{(0)})^2 - \Sigma_{\pi\pi}^\sigma(0)$, because in an OBEP model for the NN interaction the exchanged boson has nearly zero momentum. $m_\sigma^2(OBE)$ is a real value because $\Sigma_{\pi\pi}^\sigma(0)$ is real. Thus any variation on $m_\sigma(OBE)$ with respect to m_π is given by variations in $m_\sigma^{(0)}$ and $\Sigma_{\pi\pi}^\sigma(0)$:

$$\frac{\delta m_\sigma^2(OBE)}{\delta m_\pi^2} = \frac{m_\sigma^{(0)}}{m_\pi} \frac{\delta m_\sigma^{(0)}}{\delta m_\pi} - \frac{\delta \Sigma_{\pi\pi}^\sigma(0)}{\delta m_\pi^2} \quad (6)$$

and using the Gell-Mann-Oakes-Renner (GMOR) relation [31]:

$$\frac{\delta m_\sigma(OBE)}{m_\sigma(OBE)} = \nu_\sigma \frac{\delta m_q}{m_q}, \quad (7)$$

with

$$\nu_\sigma = \frac{m_\pi^2}{2m_\sigma^2(OBE)} \left[\frac{\delta (m_\sigma^{(0)})^2}{\delta m_\pi^2} - \frac{\delta \Sigma_{\pi\pi}^\sigma(0)}{\delta m_\pi^2} \right]. \quad (8)$$

We change m_π near the physical value and find the variation $\frac{\delta \Sigma_{\pi\pi}^\sigma(0)}{\delta m_\pi^2}$, which is almost constant for a small change

TABLE II. Calculations for the coefficient ν_σ , which relates the fractional change of the mass of the σ meson relevant to the OBEP model to the fractional change in the quark mass.

$m_\sigma^{(0)}$	$\frac{\delta \Sigma_{\pi\pi}^\sigma(0)}{\delta m_\pi^2}$	$\frac{\delta (m_\sigma^{(0)})^2}{\delta m_\pi^2}$	$\frac{\delta m_\sigma^2(OBE)}{\delta m_\pi^2}$	ν_σ
563.54	-0.145	2.677	2.822	0.089
600.23	-0.164	2.632	2.796	0.078
639.85	-0.189	2.576	2.765	0.068
683.46	-0.220	2.546	2.766	0.060
732.00	-0.261	2.502	2.763	0.052
790.18	-0.314	2.451	2.765	0.045
859.15	-0.389	2.401	2.790	0.038
945.43	-0.495	2.344	2.839	0.032

in m_π^2 , so we only need to find the slope of the plot $\Sigma_{\pi\pi}^\sigma(0)$ versus m_π^2 . We also need the variation of $m_\sigma^{(0)}$ with m_π near 140 MeV. For this purpose we used the NJL model, which is known to respect the chiral behaviour of QCD, including the GMOR relation. The results for all the cases in Table I are contained in Table II.

From Table II, we notice that as Λ increases (Λ increases when $m_\sigma^{(0)}$ grows) the value of $\Sigma_{\pi\pi}^\sigma(0)$ also grows and its contribution of $\frac{\delta \Sigma(0)}{\delta m_\pi^2}$ to $\frac{\delta m_\sigma^2(OBE)}{\delta m_\pi^2}$ increases. However, in practice the variation of the σ bare mass is much larger and, in total, the larger $m_\sigma^{(0)}$ the smaller the coefficient ν_σ . What is more, calculations within the Quark Meson Coupling (QMC) model tend to favour values for $m_\sigma(OBE)$ near 550 MeV (see sect. IV).

B. VARIATIONS IN m_ρ AND m_ω WITH RESPECT TO m_q

In the case of the ρ meson we have a good deal of data taken from lattice calculations in partially quenched QCD from the CP-PACS collaboration. Armour *et al.* [32] used this data in an analysis that included the leading and next-to-leading non-analytic chiral corrections to the self-energy to make an extrapolation of the mass m_ρ to the chiral limit ($m_\pi \approx 0$). At the physical value of m_π they found excellent agreement with the physical value.

The relevant self-energy diagrams for the ρ are given in Fig 2.

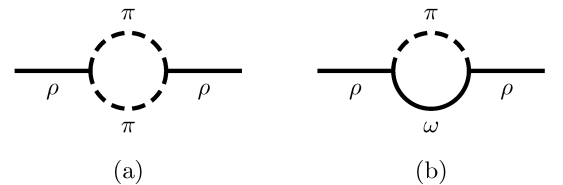


FIG. 2. Self-energy contributions for the ρ -meson.

These yield the following expressions:

$$\Sigma_{\pi\pi}^\rho = -\frac{f_{\rho\pi\pi}^2}{6\pi^2} \int_0^\infty \frac{k^4 u_{\pi\pi}^2(k) dk}{\omega_\pi(k) \left(\omega_\pi^2(k) - \frac{\mu_\pi^2}{4} \right)}, \quad (9)$$

$$\Sigma_{\pi\omega}^\rho = -\frac{g_{\omega\rho\pi}}{12\pi^2} \mu_\rho \int_0^\infty \frac{k^4 u_{\pi\omega}^2(k) dk}{\omega_\pi^2(k)}, \quad (10)$$

where $f_{\rho\pi\pi} = 6.028$ and $g_{\omega\rho\pi} = 0.016 \text{ MeV}^{-1}$. The regularization functions used in the analysis are:

$$u_{\pi\omega}(k) = \frac{\Lambda^4}{(\Lambda^2 + k^2)^2}, \quad (11)$$

$$u_{\pi\pi}(k) = \frac{\left(\Lambda^2 + \frac{\mu_\rho^2}{4} - \mu_\pi^2 \right)^2}{(\Lambda^2 + k^2)^2}, \quad (12)$$

and we use the approximations: $m_\pi \ll m_{\omega,\rho}$ and $m_\rho \approx m_\omega$.

The fit to the partially quenched lattice QCD data for the ρ meson involved a fit of the form:

$$m_\rho = \sqrt{(a_0 + a_2 m_\pi^2 + a_4 m_\pi^4)^2 + \Sigma_{TOT}}, \quad (13)$$

where $\Sigma_{TOT} = \Sigma_{\pi\pi}^\rho + \Sigma_{\pi\omega}^\rho$, and the coefficients, a_i , are: $a_0 = 832.00 \text{ MeV}$, $a_2 = 4.94 \times 10^{-4} \text{ MeV}^{-1}$, $a_4 = -6.10 \times 10^{-11} \text{ MeV}^{-3}$ and $\Lambda = 655.00 \text{ MeV}$ (up to errors). At the physical pion mass (in full QCD) this yields a value of:

$$m_\rho \approx 778 \text{ MeV}, \quad (14)$$

which shows remarkable agreement with the physical value, with a shift of only:

$$m_\rho - m_\rho^{\text{phys}} \sim 3.7 \text{ MeV}. \quad (15)$$

As in the case of the σ meson, we consider a one boson exchange potential with almost zero momentum transfer, so that $\mu_\rho \sim 0$ in the propagator of (9) (not in the regulator, because the mass that appears there is the physical mass):

$$\Sigma_{\pi\pi}^\rho = -\frac{f_{\rho\pi\pi}^2}{6\pi^2} \int_0^\infty \frac{k^4 u_{\pi\pi}^2(k) dk}{\omega_\pi^3(k)}. \quad (16)$$

This of course changes the value of m_ρ , now denoted $m_\rho(\text{OBE})$, at the physical pion mass. Indeed, in this case it is near 762 MeV. The relation between m_ρ and m_π^2 near the physical value is almost linear and it presents a slope of $\frac{\delta m_\rho(\text{OBE})}{\delta m_\pi^2} = 0.00135 \text{ MeV}^{-1}$. Following the analysis for m_σ we can relate this change with m_q in the following way:

$$\frac{\delta m_\rho(\text{OBE})}{m_\rho} = \left(\frac{m_\pi^2}{m_\rho(\text{OBE})} \frac{\delta m_\rho(\text{OBE})}{\delta m_\pi^2} \right) \frac{\delta m_q}{m_q}. \quad (17)$$

Using for $m_\rho(\text{OBE})$, henceforth simply written as m_ρ , the value 770 MeV, which is usually used in OBE models, we find:

$$\frac{\delta m_\rho}{m_\rho} = 0.034 \frac{\delta m_q}{m_q}. \quad (18)$$

The analysis for the ω meson is closely related to that of the ρ meson. However, the diagrams that contribute to the self-energy terms differ because there is no two-pion contribution because of G-parity. In addition, $\Sigma_{\pi\rho}^\omega$ is $3 \times \Sigma_{\pi\pi}^\rho$, because there are three possible $\rho - \pi$ charge combinations. For the analytic terms in the expansion we use the same coefficients (a_i) as in the case of the ρ , because the mass difference between them is only of order 10 MeV. Then the variation of m_ω with respect to m_π^2 near the physical value gives $\frac{\delta m_\omega}{\delta m_\pi^2} = 0.00096 \text{ MeV}^{-1}$, which leads to the relation:

$$\frac{\delta m_\omega}{m_\omega} = 0.024 \frac{\delta m_q}{m_q}, \quad (19)$$

where we used the physical mass for the ω , $m_\omega = 782 \text{ MeV}$ – again because that is the value typically used in a OBE potential. (The value obtained at zero momentum transfer would be 765 MeV.)

C. SUMMARY OF MESON MASS VARIATION

TABLE III. Coefficients ν_i summarising the rate of variation of the masses of the mesons used in an OBE description of the NN force with respect to quark mass - see Eq. (20).

Meson	ν (MeV)
π	0.5
η	0.012
σ_0	0.089
σ_1	0.072
ρ	0.034
ω	0.024

For the η , like the pion, we use the GMOR relation to calculate the variation within respect to u and d mass. In the case of the iso-vector scalar meson, σ_1 , which has negative G-parity and therefore does not couple to two pions, we use the NJL model - corresponding to the third column and second row of Table II, and Eq. (8) without the self-energy part. For convenience, in Table III we summarise the values of ν_i , defined as

$$\frac{\delta m_i}{m_i} = \nu_i \frac{\delta m_q}{m_q}, \quad (20)$$

which will be used below.

III. NUCLEON MASS

In order to compute the variation of nuclear binding energies with quark mass, we also need to know how the nucleon mass changes.

The variation with light quark mass is directly given by the so-called πN sigma commutator

$$\sigma_{\pi N} = m_q \langle N | \bar{q}q | N \rangle = m_q \frac{\delta m_N}{\delta m_q}, \quad (21)$$

where $\bar{q}q \equiv \bar{u}u + \bar{d}d$.

The last equality, which gives the information we need, follows from the Feynman-Hellmann theorem. A number of methods have been used to extract $\sigma_{\pi N}$ from pion-nucleon scattering data using dispersion relations, but the resulting value is still controversial.

Instead, the most reliable method seems to be to use fits to lattice QCD data for m_N as a function of m_q [33]. These fits, which build the constraints of chiral effective field theory, appear to yield very reliable values. We take the result of the latest analysis of PACS-CS data by Shanahan et. al. [35], namely $\sigma_{\pi N} = 45 \pm 6$ MeV. Thus we use:

$$\frac{\delta m_N}{m_N} = 0.048 \frac{\delta m_q}{m_q}. \quad (22)$$

IV. ${}^7\text{Li}$, ${}^{12}\text{C}$ AND ${}^{16}\text{O}$ NUCLEI

To study the effect of the quark mass variation on the single-particle energies of ${}^7\text{Li}$, ${}^{12}\text{C}$ and ${}^{16}\text{O}$ nuclei, it is highly desirable to use a nuclear model based on the quark degrees of freedom. The quark-meson coupling (QMC) model, which originated with Guichon [37] as a description of nuclear matter and was extended and improved to describe the properties of finite nuclei [38, 39], is ideal for this purpose. The successful features of the QMC model applied to various nuclear phenomena and hadronic properties in a nuclear medium, are reviewed extensively in Ref. [40]. The model has been updated to study the properties of hypernuclei [41], and neutron star structure [42, 43], where the quark structure of the nucleons and hyperons should play an important role at such high density. We calculate the change in the single-particle energies of these nuclei versus the current quark mass (m_q) and the mass of the nucleon (m_N) using the theory presented in Ref. [39] and the meson and nucleon mass changes calculated above.

In Ref. [39] the standard values used to reproduce the nuclear matter saturation properties are, $(m_N, m_\sigma, m_\omega, m_\rho) = (939, 550, 783, 770)$ MeV, with the current quark mass $m_q = 5.0$ MeV. For the calculation of finite nuclei, the ratio for the $\sigma - N$ coupling constant and the mass, (g_σ^N/m_σ) , was kept constant and fitted to the rms charge radius of ${}^{40}\text{Ca}$, $r_{ch}({}^{40}\text{Ca}) = 3.48$ fm, by adjusting $m_\sigma \rightarrow \tilde{m}_\sigma = 418$ MeV. (Note that the variation

of m_σ at fixed (g_σ^N/m_σ) has no effect on the nuclear matter properties.) To account for this, we calculate the shift in m_σ from 550 MeV for a given variation of the quark mass. This changes the ratio (g_σ^N/m_σ) and from that new value we deduce the corresponding shift in m_σ from 418 MeV, to be used in the finite nucleus calculation. First, with the nucleon mass fixed at $m_N = 939$ MeV and the variations of the meson masses, $\delta m_{\sigma, \omega, \rho}$, evaluated for quark mass variations of $\delta m_q = \pm 0.05$ and ± 0.1 MeV, we calculate the single-energies in ${}^7\text{Li}$, ${}^{12}\text{C}$ and ${}^{16}\text{O}$. Note that the very small differences for the σ and ω meson mass values used to extract the relation in terms of δm_q in II A and II B, were neglected. In addition, we also calculate the energy per nucleon (E/nucleon). The results are given in Table IV.

From Table IV we see that the absolute values of the single-particle binding energies of each nucleus decrease as the quark mass increases. This is because an increase of the quark mass leads to a significant increase of the mass of the σ meson and this reduces the attraction arising from σ meson exchange by more than the repulsion associated with the ω decreases. It is interesting to point out that a small variation of the quark mass of 0.05 MeV is reflected in a change in the single-particle energies of order of 0.1 MeV. That is, the impact is appreciable. Furthermore, we note that the binding energy per nucleon for each nucleus decreases linearly as the quark mass increases.

Next, we calculate the variation of the single-particle energies as the mass of the nucleon is varied. The results are given in Table V for the same nuclei as in Table IV. As the value of the nucleon mass increases the absolute values of the single-particle binding energies also increase. This seems to be natural, since the kinetic energy is suppressed.

It may be helpful to consider the binding energy per nucleon as a function of the quark mass. Based on the results given in Tables IV and V, and in (22); we get the following relations for each nucleus:

$$\frac{\delta |E_{{}^7\text{Li}}|/\text{nucleon}}{|E_{{}^7\text{Li}}|/\text{nucleon}} = -2.571 \frac{\delta m_q}{m_q}, \quad (23)$$

$$\frac{\delta |E_{{}^{12}\text{C}}|/\text{nucleon}}{|E_{{}^{12}\text{C}}|/\text{nucleon}} = -1.438 \frac{\delta m_q}{m_q}, \quad (24)$$

$$\frac{\delta |E_{{}^{16}\text{O}}|/\text{nucleon}}{|E_{{}^{16}\text{O}}|/\text{nucleon}} = -1.082 \frac{\delta m_q}{m_q}. \quad (25)$$

The contributions to the previous coefficients from the variation of the exchanged mesons masses are found from Table IV, and from Table V we obtain the contribution from the nucleon mass. These calculations are summarised in the following equation:

$$\frac{\delta |E_i|/\text{nucleon}}{|E_i|/\text{nucleon}} = (\nu_{\text{mesons}} + \nu_{\text{Nucleon}}) \frac{\delta m_q}{m_q},$$

with i representing each of the three nuclei we are con-

TABLE IV. Single-particle energies (in MeV) for ${}^7\text{Li}$, ${}^{12}\text{C}$ and ${}^{16}\text{O}$ nuclei versus quark mass m_q (in MeV) calculated in the quark-meson coupling (QMC) model [39]. E/nucleon stands for energy per nucleon. The standard value for the quark mass use in the QMC model is $m_q = 5.00$ MeV.

	States	m_q	4.90	4.95	5.00	5.05	5.10
${}^7\text{Li}$							
p	$1s_{1/2}$		-19.0216	-18.8657	-18.7089	-18.5522	-18.3953
	$1p_{3/2}$		-3.5945	-3.5104	-3.4267	-3.3432	-3.2602
n	$1s_{1/2}$		-18.3503	-18.2146	-18.0777	-17.9408	-17.8039
	$1p_{3/2}$		-3.2385	-3.1709	-3.1036	-3.0366	-2.9699
E/nucleon			-1.710	-1.662	-1.614	-1.566	-1.519
${}^{12}\text{C}$							
p	$1s_{1/2}$		-26.2579	-26.1110	-25.9643	-25.8179	-25.6716
	$1p_{3/2}$		-10.0448	-9.9445	-9.8444	-9.7447	-9.6453
n	$1s_{1/2}$		-29.4077	-29.2558	-29.1040	-28.9525	-28.8011
	$1p_{3/2}$		-12.9435	-12.8379	-12.7327	-12.6277	-12.5231
E/nucleon			-4.174	-4.107	-4.040	-3.974	-3.908
${}^{16}\text{O}$							
p	$1s_{1/2}$		-29.0713	-28.9260	-28.7810	-28.6362	-28.4917
	$1p_{3/2}$		-13.8605	-13.7503	-13.6405	-13.5309	-13.4217
	$1p_{1/2}$		-12.0635	-11.9647	-11.8661	-11.7679	-11.6700
n	$1s_{1/2}$		-33.0861	-32.9358	-32.7857	-32.6358	-32.4862
	$1p_{3/2}$		-17.6266	-17.5112	-17.3962	-17.2815	-17.1671
	$1p_{1/2}$		-15.8151	-15.7110	-15.6073	-15.5038	-15.4006
E/nucleon			-6.109	-6.035	-5.961	-5.888	-5.815

sidering, ν_{mesons} being described by

$$\nu_{mesons} = \frac{\delta |E_i|/\text{nucleon}}{\delta m_q} \cdot \frac{m_q}{|E_i|/\text{nucleon}},$$

and $\nu_{Nucleon}$ by

$$\nu_{Nucleon} = \frac{\delta |E_i|/\text{nucleon}}{\delta m_N} \cdot \frac{m_N}{|E_i|/\text{nucleon}} \cdot 0.048.$$

V. VARIATION IN THE ENERGIES OF THE TWO- AND THREE-NUCLEON SYSTEM WITH VARIATION IN THE MESON AND NUCLEON MASSES

To examine the variation in the binding energy of the deuteron and triton with changes in the meson and nucleon masses, we need to consider a purely One Boson Exchange (OBE) model for the nucleon-nucleon interaction. We choose to employ the OBE potential of Bryan-Scott (BS) [44], which includes the exchange of $(\pi, \eta, \sigma_0, \sigma_1, \rho, \omega)$ -mesons. To avoid the singular nature of this potential, BS introduced a monopole regularization scheme that insured that the potential is finite at the origin. With a cutoff mass of 1500 MeV this regularization is shorter in range than the range of the heaviest

of the bosons included in the potential. As a result, the medium range interaction is dominated by the σ_0, σ_1 followed by the ρ and ω exchanges.

Because of the nonlocal nature of the potential (term proportional to ∇^2), we have used the method of moments [45] to solve the Schrödinger equation for the binding energy of the deuteron and the ${}^1\text{S}_0$ amplitude. This entails expanding the radial wave function $\psi_\ell(r)$ for a given angular momentum ℓ as a linear combination of Yamaguchi [46] wave functions $\psi_\ell^{(Y)}(r; \beta_i)$ with different range parameters β_i , *i.e.* [47]

$$\psi_\ell(r) = \sum_{i=1}^n b_i^\ell \psi_\ell^{(Y)}(r; \beta_i), \quad (26)$$

where we have taken $n = 12$ and the β_i are multiples of the pion mass. The present choice for the variational wave function ensures that the correct long-range behavior of $\psi_\ell(r)$ is that defined by the asymptotic behavior of $\psi_\ell^{(Y)}(r)$. This in turn is determined by the binding energy of the deuteron or the ${}^1\text{S}_0$ anti-bound state. This procedure reduces the two-body Schrödinger equation to a set of $2n$ homogenous algebraic equations that give us the binding energy and the wave function for the deuteron to a very good approximation [45, 47].

For the ${}^1\text{S}_0$, the pole in the scattering amplitude is on the second energy sheet, and the analytic continuation of

TABLE V. Single-particle energies (in MeV) for ${}^7\text{Li}$, ${}^{12}\text{C}$ and ${}^{16}\text{O}$ nuclei versus nucleon mass m_N (in MeV), calculated in the quark-meson coupling (QMC) model [39]. E/nucleon stands for energy per nucleon. The standard value for the nucleon mass used in the QMC model is $m_N = 939.0$ MeV.

States		m_N	938.0	938.5	939.0	939.5	940.0
${}^7\text{Li}$							
p	$1s_{1/2}$		-18.6768	-18.6929	-18.7089	-18.7249	-18.7409
	$1p_{3/2}$		-3.4010	-3.4139	-3.4267	-3.4396	-3.4524
n	$1s_{1/2}$		-18.0517	-18.0647	-18.0777	-18.0907	-18.1036
	$1p_{3/2}$		-3.0830	-3.0933	-3.1036	-3.1138	-3.1241
E/nucleon			-1.600	-1.607	-1.614	-1.621	-1.628
${}^{12}\text{C}$							
p	$1s_{1/2}$		-25.9445	-25.9544	-25.9643	-25.9742	-25.9841
	$1p_{3/2}$		-9.8211	-9.8328	-9.8444	-9.8561	-9.8677
n	$1s_{1/2}$		-29.0828	-29.0934	-29.1040	-29.1147	-29.1253
	$1p_{3/2}$		-12.7076	-12.7202	-12.7327	-12.7452	-12.7577
E/nucleon			-4.022	-4.031	-4.040	-4.050	-4.059
${}^{16}\text{O}$							
p	$1s_{1/2}$		-28.7634	-28.7722	-28.7810	-28.7897	-28.7985
	$1p_{3/2}$		-13.6182	-13.6293	-13.6405	-13.6516	-13.6627
	$1p_{1/2}$		-11.8432	-11.8547	-11.8661	-11.8775	-11.8889
n	$1s_{1/2}$		-32.7665	-32.7761	-32.7857	-32.7952	-32.8048
	$1p_{3/2}$		-17.3721	-17.3841	-17.3962	-17.4082	-17.4202
	$1p_{1/2}$		-15.5826	-15.5949	-15.6073	-15.6196	-15.6319
E/nucleon			-5.941	-5.951	-5.961	-5.971	-5.981

the method of moments to the second energy sheet is not as simple, because the pole is along the negative imaginary momentum axis. However, since this anti-bound state pole is close to the zero energy ($E_P = -0.066$ MeV), we have chosen the zero energy point to reduce the Schrödinger equation using the method of moments to a set of n algebraic equations. It has been demonstrated [47] that this procedure gives a good representation of the original potential for the low energy scattering parameters. As a result we use the effective range expansion to determine the position of the anti-bound state pole in the momentum or k -plane, *i.e.* we write the on-shell ${}^1\text{S}_0$ amplitude in terms of the phase shifts δ_0 as

$$t(k) = -\frac{\hbar^2}{\pi\mu} \frac{1}{k \cot \delta_0 - ik}, \quad (27)$$

where μ is the reduced mass, and make use of the effective range expansion

$$k \cot \delta_0 = -\frac{1}{a_s} + \frac{1}{2} r_s k^2 - P_s r_s^3 k^4 + \dots, \quad (28)$$

where P_s is the shape parameter, to analytically continue the amplitude onto the second energy sheet. Since the anti-bound state is close to zero energy ($k \approx -0.04i$), we can truncate the effective range expansion to include the k^4 term. To test the accuracy of this procedure,

we compare the position of the pole on the second energy sheet for the Bryan-Scott potential by truncating at k^2 and k^4 term with the result $E_P = -0.0711531$ and -0.0711548 MeV respectively. As a result we have chosen to truncate the effective range expansion to include the k^4 term.

The use of the trial function in Eq. (26) has the added advantage of allowing us to construct an equivalent rank one separable potential, often referred to as the Unitary Pole Approximation (UPA), that has identically the same deuteron wave function as the original OBE potential [45]. After partial wave expansion, this is of the form

$$V_{\ell;\ell'}^{\text{UPA}}(k, k') = g_\ell(k) C_{\ell;\ell'} g_{\ell'}(k'), \quad (29)$$

where the form factors are directly related to the radial wave function $\psi_\ell(r)$ and the strength of the potential is adjusted to insure that the matrix element of the UPA and original OBE potential are identical at the energy of the pole in the amplitude. The same procedure is applied to the ${}^1\text{S}_0$ channel.

Having constructed a rank one separable potential equivalent to the OBE potential, we can write the Faddeev equations as a set of coupled one dimensional integral equations [48]. If one includes the ${}^1\text{S}_0$ and ${}^3\text{S}_1$ - ${}^3\text{D}_1$ nucleon-nucleon partial waves only, then the number of

TABLE VI. Variation in the position of the anti-bound state pole on the second energy sheet, the binding energy of the deuteron and triton with changes in hadron mass m_H . For the Bryan-Scott potential the position of the anti-bound state pole is $E_P = -7.1155\text{E-02}$ MeV, deuteron binding energy $E_D = 2.18365$ MeV, while the triton binding energy $E_t = 7.9131$ MeV in the UPA.

H	m_H (MeV)	$\frac{\delta E_P}{\delta m_H}$	$\frac{\delta E_D}{\delta m_H}$	$\frac{\delta E_t}{\delta m_H}$
π	138.7	2.38E-03	-0.0201	-0.0146
η	548.7	-7.40E-05	0.0019	0.0034
σ_0	550.0	-1.99E-03	-0.1026	-0.3355
σ_1	600.0	-3.09E-03	0.0486	0.0790
ρ	763.0	1.27E-04	-0.0295	-0.0517
ω	782.8	1.34E-02	0.0923	0.2776
N	938.92	2.95E-04	0.0289	0.0527

coupled integral equations reduces to five, and these can be solved for the binding energy and wave function of the triton [49].

To examine the variation in the binding energy with changes in the mass of the mesons and nucleon, we have calculated the slope of the binding energy as function of the mass at the value of the mass used in the OBE potential. In Table VI we present this variation in the energy of anti-bound state, the deuteron and triton binding energies with respect to the variation in the masses of the six bosons included in the OBE potential. We have also included the variation in the binding energies with changes in the nucleon mass m_N . Here, we note that the nucleon mass is present, not only in the kinetic energy of the two- and three-body equations, but also in the definition of the Bryan-Scott OBE potential. For the one pion exchange component, the strength of the potential is proportional to $(g_{\pi NN}/2M)^2$ which is equivalent to $(f_{\pi NN}/m_\pi)^2$ had BS used a pseudo-vector coupling in the Lagrangian. From the Goldberger-Treiman [50] relation we have that

$$\frac{g_{\pi NN}}{M} \propto \frac{g_A}{f_\pi}, \quad (30)$$

where f_π is the pion decay constant. Although g_A and f_π are dependent on the quark mass, the ratio to first order is not sensitive to variation in quark mass. This suggests that the strength of the one pion exchange component of the BS should not change with changes in the nucleon mass. Since the η is part of the same $SU(3)$ octet as the pion, one could apply the same argument the η exchange component of the OBE potential. For the scalar (σ_0 and σ_1) and vector (ρ and ω) meson exchanges, the relative strength of the central, the spin-orbit and the tensor component depend on the nucleon mass, and to that extent, we have maintained the M dependence of the OBE potential for the scalar and vector exchanges. From Table VI we observe that the variation is largest for

the σ_0 and ω , followed by the variation with the π , σ_1 , ρ and N masses, with the variation in the energy with the η mass being minimal.

A. Total variation in binding energies

From the detailed results given in Table VI and the earlier results for the variation of the meson and nucleon masses with quark mass, we can readily deduce the total variation of the deuteron and triton binding energies and the energy of the anti-bound state, E_P , with changes in the quark mass:

$$\frac{\delta E_D}{E_D} = -0.912 \frac{\delta m_q}{m_q}, \quad (31)$$

$$\frac{\delta E_t}{E_t} = -0.980 \frac{\delta m_q}{m_q} \quad (32)$$

and

$$\frac{\delta E_P}{E_P} = -2.839 \frac{\delta m_q}{m_q}. \quad (33)$$

The details for these calculations are shown in the Appendix.

The variations of the deuteron and triton binding energies given in Eqs. (31) and (32), respectively, are completely compatible with those reported by Flambaum and Wiringa [24]. In particular, the coefficients on the rhs of those equations, namely -0.91 for the deuteron and -0.98 for the triton, are very close to those reported in Ref. [24] for the AV14 potential, namely -0.84 and -0.89.

On the other hand, for the 1S_0 anti-bound state, with energy E_P , there is a significant disagreement. The sign reported above for $\delta E_P/E_P$ is negative, whereas a positive value was reported in Ref. [24]. Since Dmitriev *et al.* [24] presented an apparently general argument relating the change in the deuteron binding to that in the energy of the anti-bound state, we re-checked every term in our calculation carefully. There is no doubt that our result is correct for the model used. We note that, from Table II of Flambaum and Wiringa [24], the individual pieces of the Argonne potential do *not* respect the supposedly general result of Dmitriev *et al.* and therefore it cannot be a model independent result. We note, in particular, that the tensor force plays a significant role for the deuteron, whereas it is absent for the 1S_0 channel. Clearly, this difference for the 1S_0 anti-bound state will lead to significant changes when one computes the effect of a change in quark mass on the reaction rate for $n p \rightarrow d \gamma$.

VI. CONCLUSIONS

We have calculated the variation of the binding energy of the deuteron, triton and the 1S_0 anti-bound pole

position, as well as the binding energy per nucleon for a number of light nuclei, with respect to variations in the light (average of u and d) quark mass. The results, expressed in terms of a parameter K_A , defined by

$$\frac{\delta BE(A)}{BE(A)} = K_A \frac{\delta m_q}{m_q}, \quad (34)$$

are summarised in Table-VII. In order to determine these coefficients, we first calculated the change with quark mass of the mesons used in a typical one-boson-exchange treatment of the nucleon-nucleon force. Those results were summarised in Table III. For each nucleus we calculated the rate of change of the binding energy with respect to the mass of each meson and the mass of the nucleon itself. The values of K_A were obtained by combining the latter with the results in Table III.

TABLE VII. Coefficients K_A summarising the rate of variation of the binding energies and the 1S_0 anti-bound state pole with respect to quark mass - see Eq. (34).

Nucleus	K_A
D	-0.912
T	-0.979
E_P	-2.839
^7Li	-2.571
^{12}C	-1.438
^{16}O	-1.082

For the deuteron our result, $K_d = -0.91$, is very close to that reported by Flambaum and Wiringa [24] using the AV14 potential, namely -0.84 . Similarly for the triton, our value $K_t = -0.89$ is very close to their value of -0.98 . The closeness of these results for two rather different treatments of the NN force lends considerable confidence in their reliability. However, for the position of the 1S_0 anti-bound state our calculation differs considerably from that of Ref. [24], taking the opposite sign. This suggests that this quantity may be rather more model dependent than has been realized hitherto.

In the case of light nuclei, the binding energies reported here were calculated in the quark-meson coupling (QMC) model, a relativistic mean-field model that takes into account the self-consistent response of the internal structure of the nucleon to these mean fields. Through the self-consistency, the model yields many-body [51] or equivalently density-dependent interactions [52]. Indeed, the density dependent Skyrme forces derived from QMC have proven remarkably realistic [53]. The values of K_A deduced in this way for ^7Li , ^{12}C and ^{16}O are reported in Eqs. (24)- (25). It is interesting that the value obtained for ^7Li , namely $K_{^7\text{Li}} = -2.57$, is significantly larger than that reported in Ref. [24], namely -1.03 (AV14) and -1.50 (AV18+UIX). These authors did suggest that the uncertainty on the value of K could be as large as a factor

of two and our value is consistent at that level. Clearly, this degree of variation calls for more investigation to see whether the model dependence can be reduced.

Our study of these variations of binding energies with quark mass is, of course, motivated by the possible effects on big bang nucleosynthesis (BBN). Amongst the many challenges there, the sizeable discrepancy in the abundance of ^7Li with the latest photon-to-baryon ratio (post WMAP) is of particular interest. Figure 3 illustrates the ^7Li abundance calculated using the BBN code of Kawano [54], if one allows *only* the binding energy of the deuteron and the energy of the virtual 1S_0 state to change with quark mass. The curves correspond to the values of K_d and K_P calculated here (solid line) as well as the values used by Berengut *et al.* [55] (dashed line). The substantial difference in slope means that while a 3% shift in $\delta m_q/m_q$ would suffice to reproduce the empirical abundance using the values of Berengut *et al.*, with our values this would require a huge change in quark mass. This simple example illustrates the importance of a complete study of the BBN problem including all of the consequences of a shift of quark mass within the current approach, which we leave for future work. Finally, we

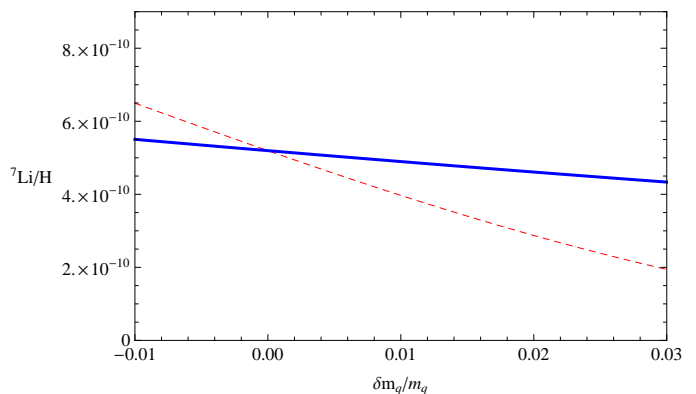


FIG. 3. (Color online) Abundance of ^7Li with respect to changes in the quark mass in $p(n, \gamma)d$ calculated in the same way as [55] (dashed-red line) and using our results for K_D and K_{E_P} (continuous-blue line).

note that while the variation of the light quark masses should be most important, it will also be necessary to take into account the effect of a corresponding change in the strange quark mass, especially now that the strange quark sigma commutator seems to be under control [56].

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APPENDIX

From Table VI we find the variations of the binding energies for the deuteron and triton (E_i with $i = D, T$), and the position of the pole for the 1S_0 anti-bound state E_P ; according to changes in the mass of the hadrons (m_H):

$$\frac{\delta E_i}{E_i} = \frac{1}{E_i} \sum_H \frac{\delta E_i}{\delta m_H} \delta m_H. \quad (35)$$

We then relate the variation of the mass of each hadron to the variation of the quark mass, as given in Eq. (20):

$$\frac{\delta m_H}{m_H} = \nu_H \frac{\delta m_q}{m_q}, \quad (36)$$

so that:

$$\delta m_H = (\nu_H \cdot m_H) \frac{\delta m_q}{m_q}. \quad (37)$$

Combining those results we finally obtain the formula that gives rise to the results in Eqs. (31), (32) and (33):

$$\frac{\delta E_i}{E_i} = \frac{1}{E_i} \sum_H \frac{\delta E_i}{\delta m_H} (\nu_H \cdot m_H) \frac{\delta m_q}{m_q}. \quad (38)$$

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